

MathFLIX CHALLENGE

Digital Roots

The **digital root** of a number is found by adding its digits, then adding the digits of any resulting number, until the final result is a single digit.

For example, the digital root of 2014 is 7 because $2 + 0 + 1 + 4 = 7$;

the digital root for 2914 is 7 because $2 + 9 + 1 + 4 = 16 = 1 + 6 = 7$

Complete the table below to find the digital roots of the numbers listed.

#	Sum of digits (single step)	Digital Root	#	Sum of digits (multiple steps)	Digital Root
11	$1 + 1$	2	59	$5 + 9 = 14 = 1 + 4$	5
101	$1 + 0 + 1$		189	$1 + 8 + 9 = 18 = 1 + 8$	
123			999		
333			1,879		
1,123			4,518		

Short-cut to finding digital roots: Casting out 9

Digital roots use MOD 9 (modular arithmetic using 9 digits) so you can cross off or “cast-off” 9s or sums of 9s with no effect on the digital root. Be careful not to cast off the last 9. The digital root cannot be 0.

Complete the table below to find the digital roots of the numbers listed. Use the “cast-off” 9s rule.

#	Short-cut (Cast out 9s.)	Digital Root	#	Short-cut (Cast out sums of 9.)	Digital Root	#	Short-cut (Digital root cannot be 0)	Digital Root
97	$9 + 7$	7	184	$1 + 8 + 4$	4	99	$9 + 9$	
908			275			999	$9 + 9 + 9$	
799			366			459		
1,998			45,182			1,836		

Usefulness of Digital Roots

You can use digital roots as a quick check for addition and multiplication computation.

Use digital roots to check this computation. Make an x through incorrect answers.

Addition Computation	Digital Root Check	Multiplication Computation	Digital Root Check
$\begin{array}{r} 44 \\ +10 \\ \hline 54 \end{array}$	$4 + 4 = 8$ $1 + 0 = 1$ $5 + 4 = 9$	$\begin{array}{r} 44 \\ \times 10 \\ \hline 440 \end{array}$	$4 + 4 = 8$ $1 + 0 = 1$ $4 + 4 = 9$
$\begin{array}{r} 235 \\ +197 \\ \hline 332 \end{array}$		$\begin{array}{r} 379 \\ \times 454 \\ \hline 172,066 \end{array}$	
$\begin{array}{r} 235,197,999 \\ +197,451,877 \\ \hline \end{array}$		$\begin{array}{r} 279 \\ \times 954 \\ \hline 272,066 \end{array}$	