

MathFLIX CHALLENGE

Happy Numbers

Numbers can have many different “personalities.” For example, some numbers are categorized as odd, even, square, cubed, prime or composite. Sophie Germain, a famous female mathematician, discovered that all numbers can be categorized as “happy” or “unhappy.” Let see how she came to her conclusion.

A happy number is one in which the **sum of each digit squared eventually ends in the number 1.**

Let's test 68

$$6^2 + 8^2$$
$$36 + 64 = 100$$
$$1^2 + 0^2 + 0^2$$
$$1 + 0 + 0 = 1$$



An unhappy number is one in which the **sum of each digit squared creates an endless loop.**

Let's test 20

$$2^2 + 0^2$$
$$4 + 0 = 4$$
$$4^2 = 16$$
$$1^2 + 6^2 =$$
$$1 + 36 = 37$$
$$3^2 + 7^2 =$$
$$9 + 49 = 58$$
$$5^2 + 8^2 = 25 + 64 = 89$$
$$8^2 + 9^2 =$$
$$64 + 81 = 145$$
$$1^2 + 4^2 + 5^2 =$$
$$1 + 16 + 25 = 42$$
$$4^2 + 2^2 =$$
$$16 + 4 = 20$$

Where did we begin?



How many of the first 100 counting numbers do you think are happy? Use Sophie Germain's method to identify the happy numbers and then circle them. Watch for patterns!

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

- If 42 is an unhappy #, what can you predict about 24? •
- Is the sum of two happy numbers always a happy #? •